Math 3Z03
Assignment #4

DUE: MONDAY, MARCH 9TH IN CLASS

SOLVE ANY 5 OF THE FOLLOWING 6 PROBLEMS:

1. Show that

$$\sqrt{2} = 1 + \cfrac{1}{2 + \frac{1}{2 + \frac{1}{2 + \cdots}}}$$

2. Prove the following formula for the Fibonacci numbers:

$$F_{n-1} = \frac{1}{\sqrt{5}} \left(\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n \right)$$

Recall that the Fibonacci numbers are defined by: $F_{n+1} = F_n + F_{n-1}$ with $F_0 = F_1 = 1$.

3. The great Persian poet *Omar Khayyam*, circa 1050-1130 found a geometric solution to the cubic equation $x^3 + a^2 x = b$ by intersecting a pair of conic sections. In modern notation, he constructed the parabola $x^2 = a\,y$ $(a > 0)$and intersected it with the circle pasing through the origin with centre $\frac{b}{2a^2}$. Show that the x coordinate of the point of intersection is a root of the given cubic.

4. (From Cardano's *Ars Magna*) An oracle ordered a prince to build a sacred building whose volume should be 400 cubits, the length being 6 cubits more than the width and the width 3 cubits more than the height. What is the height?
(*Hint:* Use Cardano's formula for solving cubic equations)

5. What are quaternions and who discovered them?. Use quaternions to show that the product of sums of four squares is a sum of four squares.
(*Lagrange* proved in 1770 that every natural number is the sum of four squares of natural numbers)

6. Solve the following problem posed by William Rowan Hamilton (1805-1865):
Find a route along the edges of a dodecahedron that passes exactly once through each vertex.

Hint: Make a cardboard model of the dodecahedron!