Math 3Z03
Assignment #2

DUE: FRIDAY, FEB. 2ND, 2015 *(Please hand it to me in class)*

SOLVE ANY 6 OF THE FOLLOWING 7 PROBLEMS:

1. (From *Chiu Chang Suan Shu* c. 400 AD). The height of a wall is 10 *chi'ih*. A pole of unknown length leans against the wall so that its top is even with the top of the wall. If the bottom of the pole is moved 1 *chi'ih* further from the wall, the pole will fall to the ground. What is the length of the pole?

2. Derive the cosine formula

$$\cos\frac{a}{R} = \cos\frac{b}{R}\cos\frac{c}{R} + \sin\frac{b}{R}\sin\frac{c}{R}\cos\alpha$$

for a geodesic triangle on a sphere of radius R, with sides a, b, c and corresponding angles α, β, γ. What happens when $R \to \infty$? (*Trigonometric formulas of this kind were known to Hindu and Arab mathematicians during the 10th century AD*).

3. Prove the following infinite product formula for π discovered by *François Vièta* (1540-1603):

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2}\,\frac{\sqrt{2+\sqrt{2}}}{2}\,\frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}\cdots$$

4. Sum the infinite series:

$$\sum_{n=1}^{\infty}\frac{n}{2^n}$$

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(This was calculated by *Richard Suiseth* in a book from around 1350, called the *Liber Calculationum.* At about the same time *Nicholas Orseme* (1340-1382) summed this and similar series by geometric arguments)

5. Demonstrate Leibniz's result that

$$\sum_{n=2}^{\infty}\sum_{p=2}^{\infty}\frac{1}{n^p} = 1$$

6. Prove that e (the base of the natural logarithm) is an irrational number. (*e is in fact, transcendental (who proved that first?)*)

7. Define the *cross-ratio* of four points on a line and prove that the cross-ratio is preserved under a projection from a point to any other line.