

Math 3Z03
Assignment #1

DUE: MONDAY, JANUARY 19TH, 2015 (*Please hand it to me in class*)

SOLVE ANY 6 OF THE FOLLOWING 7 PROBLEMS:

1. Here is a conundrum about Diophantus: *God granted him to be a boy for the sixth part of his life, and adding a twelfth part to this, He clothed his cheeks with down. He lit him the light of wedlock after a seventh part, and five years after his marriage He granted him a son. Alas! late-born wretched child; after attaining the measure of half his father's life, chill Fate took him. After consoling his grief by this science of numbers for four years he ended his life.*
How old was Diophantus when he got married and at what age did he die?

2. What is a perfect number? Show that the reciprocals of all the positive divisors of a perfect number add up to two.

3. In *Almagest* Ptolemy proved the following result about concyclic quadrilaterals known today as "Ptolemy's theorem." If $ABCD$ is a convex quadrilateral inscribed in a circle then the product of the diagonals is equal to the sum of the two products of the two pairs of opposite sides. Prove it!

4. *Alcuin of York, 775* A hundred bushels of grain are distributed among one hundred people in such a way that each man receives three bushels, each woman two bushels and each child half a bushel. How many men, women and children are there?
(Note: There is more than one answer!)

5. Show that

$$\phi = \frac{1 + \sqrt{5}}{2} = 1 + \frac{1}{1 + \frac{1}{1 + \dots}}$$

(An expression as above is called a continued fraction.)

6. Does the number ϕ mean anything to you? Prove that ϕ is irrational.

7. The symbol of the Pythagorean brotherhood was the mystical pentagram, the 5-pointed star formed by the five diagonals of a regular pentagon. Prove that each of the sides of the pentagram divides the the 2 sides that it intersects in a golden ratio. What is the golden ratio? Can you find other golden ratios in the pentagram?

