Math 3D03
Assignment #4

Due: Tuesday, March 18th, 2014 in class (at the beginning of the lecture period)

You can use software to check your answers but you are required to show your calculations

1. Do problem 30.26 on page 1216 in the textbook.

2. Do problem 30.36 on page 1218 of the textbook

3. A point starts at the origin on the real line and takes steps of length δ with probability $p > 0$ to the right and with probability $q = 1 - p$ to the left. Assuming that the steps are independent find the expected value of the **squared** distance from the origin after n steps.

4. A model for the movement of a stock price supposes that if the present price is S then after one period, it will either go up to uS with probability p or go down to dS with probability $1-p$. Assuming that successive movements are independent, approximate the probability that the stock price will be up by at least 10% after the next 1000 periods for $u = 1.02$, $d = 0.95$ and $p = 0.6$.

5. Let X be a random variable having the χ^2 distribution with n degrees of freedom and let Z be a standard normal variate. Assuming X and Z are independent, compute the probability density function of

$$T = \frac{Z}{\sqrt{\frac{X}{n}}}$$

6. *(bonus question)*
Let r (*the interest rate*), σ (*the volatility*), K (*strike price*) and T (*time to expiration*) be positive constants. Suppose that the price of a stock at time t can be expressed as

$$S(t) = S(0)\exp\left((r - \frac{1}{2}\sigma^2)t + \sigma\sqrt{t}\,Z\right)$$

where $S(0)$ (*present value of the stock*) is a positive number and Z is the standard normal.

Let I be the indicator variable for the event that $S(T) \geq K$ (*to be in the money*), i.e. $I = 1$ if $S(T) \geq K$ and $I = 0$ otherwise.

(i) Show that

$$\mathbb{E}[I] = \Phi(\omega - \sigma\sqrt{T}) \qquad \text{and} \qquad \mathbb{E}[IS(T)] = S(0)e^{rT}\Phi(\omega)$$

where

$$\omega = \frac{rT + \frac{1}{2}\sigma^2 T - log(\frac{K}{S(0)})}{\sigma\sqrt{T}}$$

(ii) Compute (*Black-Scholes formula for a simple European Call Option*):

$$C(S(0), T, K, r, \sigma) = e^{-rT}\,\mathbb{E}[(S(T) - K)^+] = S(0)\,\Phi(\omega) - e^{-rT}\,K\,\Phi(\omega - \sigma\sqrt{T})$$

where $x^+ = max(x, 0) = \frac{1}{2}(|x| + x)$ for any real number x.