

**Math 3D03**  
**Assignment #1**

DUE: TUESDAY, JANUARY 20TH, 2015 IN CLASS (PLEASE HAND IT TO ME AT THE BEGINNING OF THE LECTURE PERIOD)

*Note: You are required to show your calculations. You can use symbolic software **only** to check your answers.*

1. Compute all values of

$$i^{(i^i)} \quad \text{and} \quad (i^i)^i$$

2. Classify all the singular points of the following functions:

$$(a) f(z) = \frac{\pi z}{\sin(\pi z)} \quad (b) f(z) = \frac{z-2}{z^2} \sinh \frac{1}{1-z} \quad (c) f(z) = \frac{e^{\frac{1}{z}}}{1-z}$$

3. Compute the complete Taylor, respectively Laurent series expansion and the region of convergence of the following functions around the point  $z = 0$ :

$$(a) f(z) = \frac{1}{2i} \log \left( \frac{1+iz}{1-iz} \right) \quad (b) f(z) = \frac{e^{\frac{1}{z}}}{1-z}$$

4. Evaluate the following complex contour integrals:

$$(a) \oint_C \frac{dz}{1-z^4} \quad (b) \oint_C \frac{e^{iz} dz}{1+z^2} \quad (c) \oint_C \frac{z^3 dz}{(z-2)^2(z^2+4)}$$

where  $C$  is the ellipse defined by:  $3x^2 + 4y^2 = 10^{10}$

5. Compute the coefficient of  $z^3$  in the power series expansion (around  $z = 0$ ) of  $(T(z))^4$ , where

$$T(z) = \frac{z}{1-e^{-z}}$$