

Math 3Q: Numerical interpolation and approximation theory

Summary of Topics for Final Exam 2009

(The chapter references refer to the main textbook by Grasselli & Pelinovsky. You should also consult the course notes, as some topics are not covered in the textbook.)

1. Introduction (Chapter 1, [A guide to Matlab](#), 2.2–2.4)

- (a) Basic definitions.
- (b) Floating point arithmetic.
- (c) Approximation and error in computation. Precision, accuracy, forward error, backward error, conditioning, truncation and rounding error.
- (d) Introduction to `matlab`.
- (e) Solution of systems of linear equations (review of 2T material). Matrix and vector norms, condition number of a matrix.

2. Solution of nonlinear equations (Chapter 8)

- (a) One-dimensional root finding: Newton–Raphson, secant method.
- (b) Multi-dimensional root finding: Newton–Raphson, Broyden’s method.
- (c) One-dimensional minimization: critical points, golden-section search.
- (d) Multi-dimensional minimization: critical points and Hessian test, steepest descent.

3. Interpolation (Chapter 5.1–5.5, 11.1–11.2, 12.1–12.2)

- (a) Properties of polynomials and polynomial interpolation.
- (b) Vandermonde, Lagrange, and Newton interpolation.
- (c) Error analysis. Convergence rates, Runge phenomenon, problems with evenly spaced points, Chebyshev interpolation.
- (d) Trigonometric interpolation.
- (e) Spline interpolation.

4. Approximation theory and curve fitting (3.1–3.2, 5.6–5.7)

- (a) Best approximations and orthogonal projections.
- (b) Systems of orthogonal polynomials, e.g. Legendre polynomials.
- (c) Finding best approximations: least squares.
- (d) Continuous approximation.

5. Numerical Differentiation and Integration (Chapter 6)

- (a) Finite difference approximation to derivatives, error analysis.
- (b) Complex step derivative.
- (c) Error analysis: truncation and roundoff error.
- (d) Richardson extrapolation.
- (e) Spectral differentiation.
- (f) Numerical quadrature: Newton–Cotes quadrature and their error analysis.
- (g) Numerical quadrature using Chebyshev points: Clenshaw–Curtis quadrature, spectral convergence of trapezoid rule for periodic integrands.
- (h) Gaussian quadratures.
- (i) Romberg integration (Richardson Extrapolation applied to quadrature formulas).