Math 3Q03 **Numerical interpolation and approximation theory** Fall 2009

Dr. N. Kevlahan
Office: HH 324, Tel: x23412
Email: kevlahan@mcmaster.ca
Home page: www.math.mcmaster.ca/kevla/teaching/3q3/3q3-home.html
Office hours: Tuesdays 14:30-15:30

Purpose of the course

This course discusses both theoretical and practical aspects of numerical interpolation and approximation. Such techniques form the core of Numerical Analysis and are the basis for solution of many important problems. We review the relevant mathematical theory and show how it can be used to construct practical algorithms. These algorithms are implemented and tested in `matlab`.

Our focus is on applications to numerical differentiation and integration of functions. However, we also review certain related special topics such as Galerkin and wavelet approximation theory.

For an excellent explanation of what numerical analysis *really is* read Trefethen's The Definition of Numerical analysis.

Topics

The chapter references refer to the main textbook by Grasselli & Pelinovsky.

1. **Introduction (Chapter 1, A guide to Matlab, 2.2–2.4, Chapter 8)**
   (a) Basic definitions.
   (b) Introduction to `matlab`.
   (c) Solution of systems of linear equations (review of 2T material).
   (d) Solution of nonlinear equations (root finding and minimization of a function).
2. **Interpolation (Chapter 5.1–5.5, 11.1–11.2, 12.1–12.2)**
   (a) Properties of polynomials and polynomial interpolation.
   (b) Vandermonde, Lagrange & Newton interpolation.
   (c) Error analysis: Runge phenomenon.
   (d) Trigonometric interpolation, error analysis.
   (e) Spline interpolation.
   (f) Hermite interpolation
3. **Approximation (3.1–3.2, 5.6–5.7)**
   (a) Best approximations and orthogonal projections.
   (b) Systems of orthogonal polynomials.
   (c) Finding best approximations.
4. **Numerical Differentiation and Integration (Chapter 6)**
   (a) Finite difference approximation to derivatives, error analysis.
   (b) Richardson extrapolation.
   (c) Numerical quadrature, error analysis.
   (d) Spectral differentiation.

(e) Gaussian quadratures.

5. **A brief overview of numerical approximation for the solution of differential equations (Chapters 9 and 10 and course notes):**

   (a) Initial value problems for ODEs.

   (b) Boundary value problems for ODEs and PDEs.

   (c) Convergence, consistency, stability, Lax's equivalence theorem.

Textbooks

Primary:
M. R. Grasselli & D. E. Pelinovsky, *Numerical Mathematics*, Jones & Bartlett (2008).

Supplemental References:
C. F. Gerald & P. O. Wheatley, *Applied numerical analysis*, Pearson, (2004)
Hunt et al. *A guide to Matlab* (2001) link to e-book.

Software

All computational examples will be presented using MATLAB. This software is available on the computers in the computer lab (Mondays 12:20-13:20 in BSB 244). Lab hours are reserved for unsupervised work with computer-based assignments. Unless they are reserved for large-class tutorials, students should be also able to work in the computer labs outside our allocated time.

Students are encouraged to purchase *The Student Edition of MATLAB* to be able to work with MATLAB at home.

I will give a short introduction to MATLAB during the course and more detail is given is the supplementary reference *A guide to Matlab*.

Prerequisites

Advanced Calculus (MATH 2A03 or 2X03) and Numerical Algebra (MATH 2T03).

Evaluation

There will be five assignments, one mid-term test, and a final exam.

Assignments

Five assignments will be given and marked for credit. Each assignment will have a significant Matlab component. Assignments are to be completed using the template available at and emailed to kevlahan@mcmaster.ca by midnight on the due date. No late assignments will be accepted. Solutions to assignments and the test will be posted on the course webpage. The tentative assignment schedule is as follows:

Assignment given	**Assignment due**
September 15	September 29
September 29	October 13
October 20	November 3
November 3	November 17
November 17	December 1

Test

There will be one 50 minute in-class test on **Friday 16 October 12:30-13:20** that will cover analytical issues only (no programming). Only the McMaster standard calculator Casio fx-991 will be allowed during the quiz.

Final exam

There will be a three-hour final examination during the December examination period. Only standard McMaster University calculators (Casio FX-991) may be used in the final examination.

Grading system

The final mark will be calculated as follows:

Homework	40%
Test	10%
Final exam	50%

I reserve the right to change the weight of any portion of this marking scheme. If changes are made, your grade will be calculated using the original weightings and the new weightings, and you will be given the higher of the two grades. At the end of the course the grades may be adjusted, but this can only increase your grade and will be done uniformly. I will use the grade equivalence chart in the university calendar to convert between letter grades, grade points and percentages.

Official notices

Excused Absences

Exemptions from the assignments or tests for valid reasons are possible, but must be requested through the office of the Associate Dean of the Faculty that you are registered with. In the event of an exemption, no make up test or assignment will be administered, but your course grade will be re-weighted by increasing the weight of the final examination to compensate for the missed test or assignment.

Academic Integrity

You are expected to exhibit honesty and use ethical behaviour in all aspects of the learning process. Academic credentials you earn are rooted in principles of honesty and academic integrity.

Academic dishonesty is to knowingly act or fail to act in a way that results or could result in unearned academic credit or advantage. This behaviour can result in serious consequences, e.g. the grade of zero on an assignment, loss of credit with a notation on the transcript (notation reads: Grade of F assigned for academic dishonesty), and/or suspension or expulsion from the university.

It is your responsibility to understand what constitutes academic dishonesty. For information on the various types of academic dishonesty please refer to the Academic Integrity Policy, located at
http://www.mcmaster.ca/academicintegrity

The following illustrates only three forms of academic dishonesty:

1. Plagiarism, e.g. the submission of work that is not one's own or for which other credit has been obtained.
2. Improper collaboration in group work.
3. Copying or using unauthorized aids in tests and examinations.