Assignment 6 **MATH 2T** **24 March 2011**

This assignment is due in the Math 2T locker in the basement of Hamilton Hall by 15:00 on Thursday 7 April.

Review exercises NOT TO BE HANDED IN:

1. (a) Explain the relationship between simultaneous iteration and power iteration.
   (b) Why is simultaneous iteration numerically unstable?
   (c) How can this instability be eliminated?
2. How can the convergence of power iteration be improved?
3. True or False: The condition number of matrix with respect to solving linear systems also determines the conditioning of its eigenvalues.
4. In using QR iteration for computing the eigenvalues of a matrix, why is the matrix usually first reduced to some simpler form (such as Hessenberg or tridiagonal)?
5. Given a real symmetric square matrix A, what method would you use to compute the following?
   (a) The smallest eigenvalue of A.
   (b) The largest eigenvalue of A.
   (c) The eigenvalue of A closest to some specified number β.
   (d) All of the eigenvalues of A.

Exercises TO BE HANDED IN: (for computer problems, an annotated *shortened* computer printout is appropriate.)

[4] 1. (a) Explain how Rayleigh quotient iteration improves the convergence rate of inverse iteration. What is its relationship to least squares approximation?

[6] (b) Apply one Rayleigh quotient iteration (by hand) with initial guess $x_0 = [0.75, 1]^T$ to the matrix

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}.$$

[2] (c) If an approximate eigenvalue has an absolute error of 0.1 after one Rayleigh quotient iteration, what (roughly) would you expect the error to be after three iterations? Justify your answer.

[6] 2. (a) Write a `matlab` routine that computes the roots of a general nth-degree polynomial (assume coefficient of highest power term is one) by forming the *companion matrix* and then using the matlab routine `eig` to compute its eigenvalues.

[4] (b) Apply your routine to calculate the roots of the polynomial

$$p(t) = 24 - 40t + 35t^2 - 13t^3 + 2t^4 + 5t^7 - 2t^8 + 10t^9 + t^{10}$$

(You can check your results using the `matlab` command `roots`.)

[2] (c) Use your result from (b) to find the 2-norm condition number of the companion matrix.

[8] 3. Question 10, chapter 4 from the textbook. Note that there is a typo in the text: the actual eigenvalues of matrix A are $\{7.49, 1.95, -1.44\}$.

[8] 4. Question 13, chapter 4 from the textbook. Test your function on the matrix (4.73) on p 217 of the textbook by comparing the results to those generated by the built-in `matlab` function `svd`.

Total: 40