Math 2T — 2011

Summary of topics for the final exam

After each subject heading I have given some key words and indicated in brackets the relevant sections of the textbook. Please note that the course is defined by the *lectures*: you should study your lecture notes carefully since not everything is covered to the same depth in the textbook.

1. **Approximations in scientific computation.** Absolute error, relative error, forward error, backward error, rounding error, truncation error, optimal stepsize. Sensitivity, conditioning and stability (1.8).
2. **Floating point arithmetic.** Floating point number system, floating point arithmetic, cancellation, machine precision. (1.7).
3. **Sensitivity and conditioning for matrices.** Error bounds, vector norms, matrix norms (2.4).
4. **Direct solution of linear systems.** Triangular systems (forward and backwards substitution), Gaussian elimination, pivoting, partial pivoting, LU factorization, computational complexity ($O(2/3n^3)$ for LU factorization) (2.5).
5. **Special linear systems.** Symmetric, positive definite, banded, sparse. Cholesky factorization of symmetric positive definite systems (2.5).
6. **Iterative methods for linear systems.** Jacobi, Gauss–Seidel, SOR. Convergence criteria in terms of matrix norms and spectral radius, convergence rate and constant, computational complexity ($O(2n^2)$ per iteration for Jacobi and Gauss–Seidel), optimal choice of relaxation parameter for SOR (2.6).
7. **Least squares problems and orthogonality.** Existence and uniqueness, sensitivity and conditioning, normal equations, orthogonalization, QR factorization, Householder transformation, Gram–Schmidt orthogonalization, ill-conditioned problems (3.1-3.6).
8. **Singular value decomposition.** Definition, properties, numerical rank, application to rank-deficient and ill-conditioned systems, computation of 2-norms, pseudo-inverse, lower rank approximation (4.5).
9. **Eigenvalues and eigenvectors.** Existence and uniqueness, diagonalizability, localizing eigenvalues (i.e. Gershgorin's theorem), spectrum and spectral radius, sensitivity and conditioning, problem transformations, Schur form (4.1, 4.2).
10. **Computation of eigenpairs** Problems with finding roots of characteristic polynomial, power iteration, inverse iteration, shifts (eigenvalue closest to a given number and acceleration), Rayleigh quotient iteration, deflation, simultaneous iteration, QR iteration, QR iteration with shifts, Hessenberg reduction (4.3, 4.4).

The exam

The exam will be held **14:00-17:00** on **Tuesday 12 April** in **MDCL-1102**.

The exam consists of seven multi-part questions (similar to the test questions) and is three hours in duration. You should bring the standard McMaster Casio calculator, but you cannot bring any other material. Questions will *not* require `matlab`. I will hold two special office hours Thursday 7 April from 11:00-12:00 and Friday 8 April from 13:00-14:00. (I will not be available Monday 11 April.)

Hint: you should practice applying the algorithms (e.g. Gaussian elimination with partial pivoting, Cholesky factorization, Jacobi, Gauss–Seidel, SOR, QR decomposition using Householder transformations, least squares, power iteration) to small matrices.