Math 2T3 **Numerical Analysis I** Winter 2011

Dr. N. Kevlahan

Lectures: Tuesdays, Wednesdays and Fridays 9:30-10:20 in Hamilton Hall 104
Tutorials: Wednesday 12:30-13:20 in BSB 241 (computer lab)

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Purpose of the course

Most problems in physics, engineering and applied mathematics are formulated in terms of equations (e.g. ordinary differential, partial differential, or stochastic) which cannot be solved analytically. However, one can almost always reduce this one difficult problem to many simple problems which can be solved numerically on a computer. More precisely, we reduce the problem to a large system of linear algebraic equations.

For an excellent explanation of what numerical analysis *really* is read Trefethen's The Definition of Numerical analysis. For a nice introduction to Numerical Analysis, I suggest reading Trefethen's entry in the *Princeton Companion to Mathematics*.

This course will teach you efficient methods for solving very large systems of linear algebraic equations. With this knowledge as a foundation you will be able to easily tackle any advanced course in scientific computation or numerical analysis. You will also learn the basic mathematical concepts of numerical error analysis (i.e. sensitivity, conditioning and accuracy) and estimation of computational complexity, which are essential for developing a useful numerical algorithm.

The course material will be a mix of applied problems, theoretical analysis, algorithm development, and practical programming. Matlab will be the computer language used for all examples and assignments, as it has been developed explicitly for numerical linear algebra.

Text

The primary text for the course is:
M.R. Grasselli & D.E. Pelinovsky (2008) *Numerical mathematics*, Jones & Bartlett.

The supplementary texts for the course are:
M.T. Heath (2002) *Scientific computing: an introductory survey*, second edition, McGraw-Hill.

R. Pratap (2002) *Getting started with matlab: a quick introduction for scientists and engineers*, Oxford University Press.

A guide to Matlab (e-book)

Software

Student edition of Matlab. We highly recommend that you buy this software. Matlab is an essential part of this course, and will be very useful in any future mathematically intensive courses. Of course, you will also be able to use Matlab in the University's computer labs.

Outline

The course is organized as follows (note that timings and content are tentative):

1. **Introduction to scientific computation using Matlab** (`m-file` tutorials from website, A guide to Matlab, 4 lectures)

(a) The `matlab` session.

(b) Using `matlab` help (e.g. built-in functions).

(c) Working with matrices and vectors.

(d) Advanced data structures: structure arrays, field arrays, cells.

(e) Creating and printing figures.

(f) Basic control structures (sequential, repetition, selection).

(g) Editing, saving and executing scripts.

(h) Functions, sub functions and cell mode.

(i) Some useful built-in functions.

2. **Numerical solution of linear systems of equations** (Chapters 1 and 2, 12 lectures)

   - Floating pointing point arithmetic. (Chapter 1)
   - Finite dimensional vectors spaces, linear maps, vector and matrix norms, sensitivity and conditioning, error bounds. (§2.1-2.4)
   - Direct methods: LU factorization, Cholesky factorization, computational complexity. (§2.5, §2.7)
   - Iterative methods for solving linear systems: Jacobi, Gauss-Seidel, SOR. (§2.6)

3. **Nonlinear equations (root finding and minimization of a function)** (Chapter 8, 3 lectures)

   (a) Interval bisection,

   (b) Fixed-point iteration.

   (c) Newton's method.

   (d) Multi-dimensions.

4. **Orthogonality and data fitting by least squares** (Chapter 3, 8 lectures)

   - Least-squares method, data fitting. (§3.4)
   - Orthogonal projections (§3.2)
   - QR factorization. (§3.3)
   - Use of singular value decomposition in least squares.

5. **Eigenvalues and eigenvectors** (Chapter 4, 10 lectures)

   (a) Basic properties. (§4.1–4.5)

   (b) Power iteration, Rayleigh quotient iteration, deflation. (§4.6)

   (c) Simultaneous iteration, orthogonal iteration, QR iteration, Hessenberg reduction. (§4.7)

   (d) Singular Value Decomposition (§4.8)

Evaluation

There will be six bi-weekly assignments, one mid-term test, and a final exam.

Assignments

Six problem sheets will be given and marked for credit. Each assignment will have a significant Matlab component. Assignments are to be dropped into the Math 2T3 locker in the basement of the Hamilton Hall by 15:00 on the due date. No late assignments will be accepted. Solutions to assignments and the test will be posted on the course webpage. The assignment tentative schedule is as follows:

Assignment given	**Assignment due**
January 4	January 18
January 20	February 3
February 3	February 17
February 17	March 10
March 10	March 24
March 24	April 7

Test

There will be one 50 minute test during the **regular class time**:

Friday 18 February

Final exam

There will be a three-hour final examination during the April examination period. Only standard McMaster University calculators (Casio FX-991) may be used in the final examination.

Grading system

The final mark will be calculated as follows:

Homework	35%
Test	15%
Final exam	50%

I reserve the right to change the weight of any portion of this marking scheme. If changes are made, your grade will be calculated using the original weightings and the new weightings, and you will be given the higher of the two grades. At the end of the course the grades may be adjusted but this can only increase your grade and will be done uniformly. I will use the grade equivalence chart in the university calendar to convert between letter grades, grade points and percentages.

Official notices

Excused Absences Exemptions from the assignments or tests for valid reasons are possible, but must be requested through the office of the Associate Dean of the Faculty that you are registered with. In the event of an exemption, no make up test or assignment will be administered, but your course grade will be re-weighted by increasing the weight of the final examination to compensate for the missed test or assignment.

Academic Integrity You are expected to exhibit honesty and use ethical behaviour in all aspects of the learning process. Academic credentials you earn are rooted in principles of honesty and academic integrity.

Academic dishonesty is to knowingly act or fail to act in a way that results or could result in unearned academic credit or advantage. This behaviour can result in serious consequences, e.g. the grade of zero on an assignment, loss of credit with a notation on the transcript (notation reads: Grade of F assigned for academic dishonesty), and/or suspension or expulsion from the university.

It is your responsibility to understand what constitutes academic dishonesty. For information on the various types of academic dishonesty please refer to the Academic Integrity Policy, located at:

http://www.mcmaster.ca/academicintegrity
The following illustrates only three forms of academic dishonesty:

1. Plagiarism, e.g. the submission of work that is not ones own or for which other credit has been obtained.

2. Improper collaboration in group work.

3. Copying or using unauthorized aids in tests and examinations.

Attention is drawn to the "Statement on Academic Ethics" and "Senate Resolutions on Academic Dishonesty" as found in the Senate Policy Statements distributed at Registration and available in the Senate Office. Any student who infringes on one of these resolutions will be treated according to the published policy. In particular, it is expected that the assignments shall be done and submitted as individual work. Students may discuss general problems or approaches, but the final solution must be a result of the student's own effort.

The Faculty of Science is concerned with ensuring an environment that is free of all adverse discrimination. If there is a problem that cannot be resolved by discussion among the persons concerned, individuals are reminded that they should contact their Department Chair, the Sexual Harassment Office or the Human Rights Consultant, as soon as possible.