Vortices for computing: the engines of turbulence simulation

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Outline

Vorticity-based mathematical representation

- Since work of Helmholtz (1858), the primary description.
- Controversy: Bertrand rejected Helmholtz's interpretation of $1/2\nabla \times \mathbf{u}$ as rotation velocity of a fluid element!
- Production of vorticity at a boundary remains a tricky issue... Helmholtz considered vortex motion in an infinite fluid.

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Important vorticity-based mathematical results

Vortex motion controls flow properties

- Helmholtz's theorem, Kelvin's Circulation theorem, helicity conservation theorem.
- Force on an obstacle:

$$\mathbf{F}(t) = -\frac{1}{2}\frac{d}{dt}\int_{V_\infty} \mathbf{x} \times \boldsymbol{\omega} \, \mathrm{d}V$$

- Far field sound:

$$p_F = -\frac{\varepsilon^{(1)}(t_r)}{15\pi c^2 r} - \frac{\rho_0}{c^2} Q_{ij}^{(3)}(t_r)\frac{x_i x_j}{r^3} + \frac{\rho_0}{c^3} Q_{ijk}^{(4)}(t_r)\frac{x_i x_j x_k}{r^4} + \cdots$$

where the Q's are moments of the vorticity distribution.

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Vortex-based flow descriptions

- Helmholtz showed that a mixing layer (i.e. unstable vortex sheet) is the driving force of a organ pipe.
- First explanation of flow dynamics using coherent vortices.
- Lift generated by a wing can be explained by a system of line vortices.

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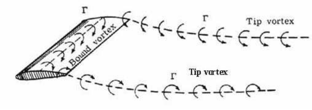

Vortex dynamics qualitatively describes fluid flow

It is therefore natural to try to use vorticity and vortices as the basis for numerical simulations of fluid flow. . .

Vortex-based numerical methods

Vorticity and vortices can also be used to calculate the flow.

1. Discretize the vorticity field.
2. Decompose the flow in coherent vortices, either instantaneously or statistically.

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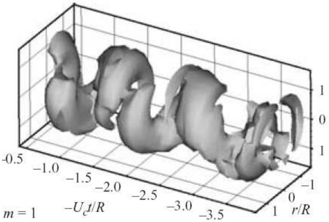

Helical coherent vortex in jet from first POD mode (Iqbal & Thomas 2007).

Vortex methods

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- Discretize the circulation of vorticity field onto N vortex particles.
- Vorticity equation integrated in two steps:
  1. Inviscid step Lagrangian advection by velocity field of other particles (Kelvin's Circulation Theorem).
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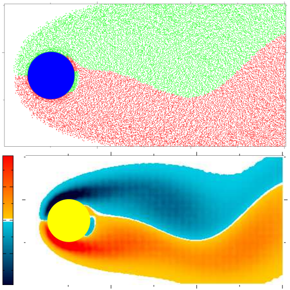

(Protas & Wesfreid 2002)

Coherent Vortex Simulation

- Use wavelet denoising to decompose flow into incoherent part and coherent part (the rest).
- Coherent part corresponds to a tiny portion ($< 1\%$) of total modes and is multiscale.
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Generalization of qualitative coherent vortex models for flow visualization.

(See Farge, Schneider & Kevlahan 1999)

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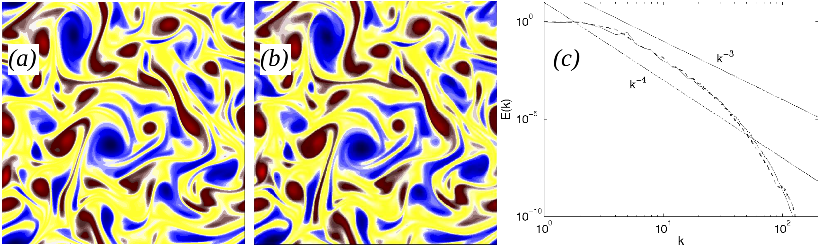

Vorticity field of 2-D turbulence at $Re = 40\,400$. (a) 263 169 Fourier modes using the pseudo-spectral method, (b) 7 895 coherent wavelet modes, (c) energy spectra: - - -, wavelet, — pseudo-spectral.

2D fluid–structure interaction: moving cylinder, $Re = 100$

Computational vortices reveal turbulence structure

Can we construct a dynamical coherent vortex model of fully developed homogeneous turbulence?

Intermittency and turbulence

- The active regions of turbulence are distributed inhomogeneously in space and time.
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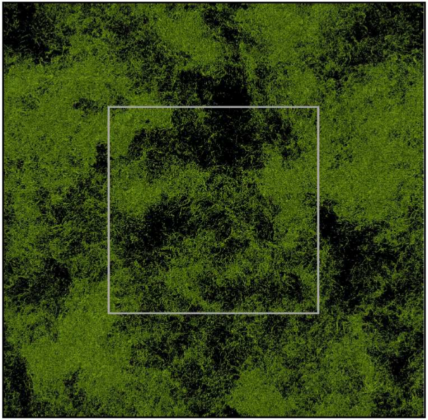

DNS of homogeneous isotropic turbulence at $\mathrm{Re}_\lambda = 1\,217$ (Yokokawa et al. 2002). Active regions are intermittent (and fractal?).

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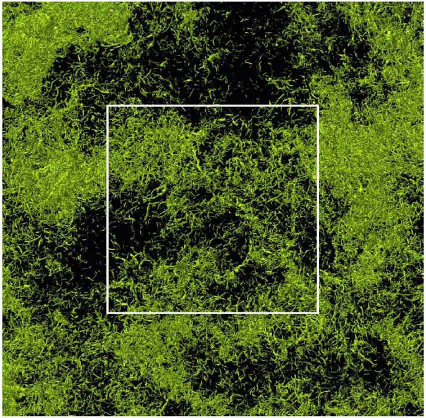

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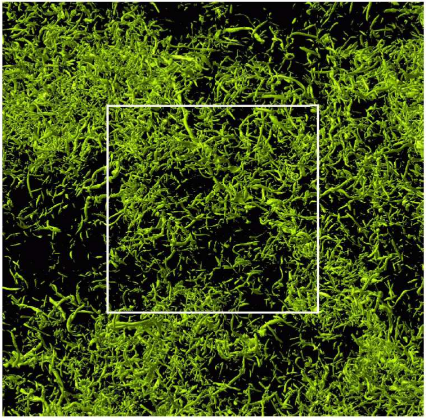

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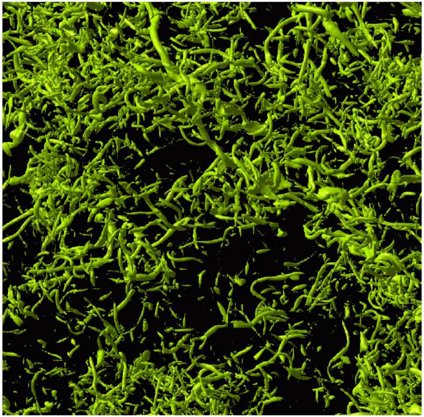

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Mathematical estimates of number of turbulence modes

- Foias & Prodi (1967) conjectured that solutions of the Navier–Stokes equations are determined uniquely by a finite number of spatial modes.
- Friz & Robinson (2001) proved this conjecture for stationary periodic 2D turbulence.
- Jones & Titi (1993) found an upper bound on the number of spatial Fourier required to represent 2D periodic turbulence of $O(Re^2)$.
- Galdi (2006) extended this result to 3D flow past bluff bodies.

Computational complexity of turbulence simulations

- Assuming homogeneity, the spatial computational complexity of turbulence scales like $Re^{9/4}$ (or Re^{1} in 2D).
- Similarly, space–time computational complexity scales like Re^{3} (or $Re^{3/2}$ in 2D).
- Yakhot & Sreenivasan recently claimed it is even worse: Re^{4}.
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Questions

- What is the actual scaling of spatial degrees of freedom with Reynolds number, Re^{β}?
- What is the actual scaling of space-time degrees of freedom with Reynolds number, Re^{α}?
- Is turbulence more intermittent in space or time?
- What is the fractal dimension of the active regions of the flow? (Assuming the $\beta-$model.)
- Do coherent vortices provide an effective reduced dynamical model of two- and three-dimensional homogeneous turbulence?

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Numerical estimation of space-time modes

- Use a simultaneous space–time adaptive wavelet solver.
- Take the number of active space–time wavelet modes as an upper bound on the number of space–time degrees of freedom.
- Consider periodic unforced turbulence.
- Perform a sequence of simulations over a wide range of Reynolds numbers.

Space–time adaptive wavelet turbulence calculation

Advantages

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- Global error control in time.
- Local time step.
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- Number of grid points is an approximation to the number of space–time degrees of freedom in the flow.

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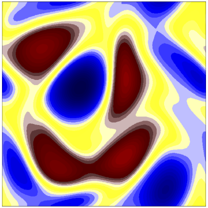
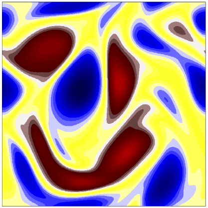
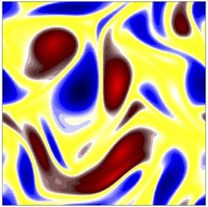
Vorticity at $t = 126$



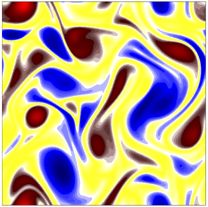
$Re = 1\,260$



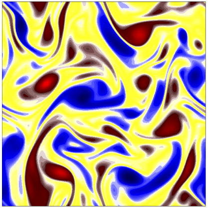
$Re = 2\,530$



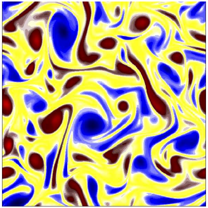
$Re = 5\,050$



$Re = 10\,100$

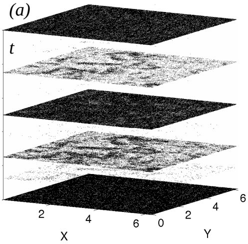


$Re = 20\,200$

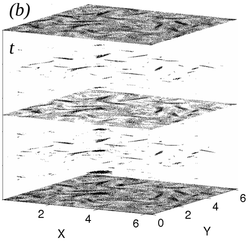


$Re = 40\,400$

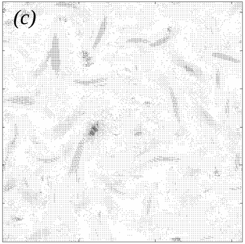
Adaptive wavelet grids at $\mathrm{Re} = 40\,400$



$t \in [0, 2.1]$



$t \in [123.8, 126.0]$



Spatial grid only at $t = 126.0$

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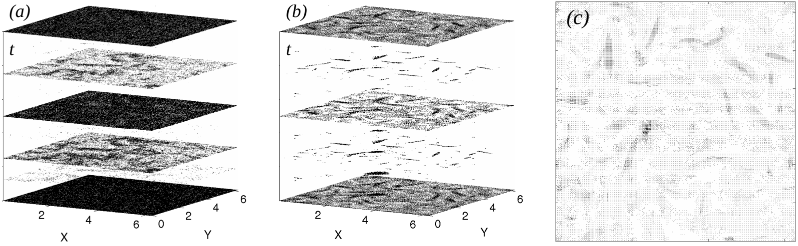
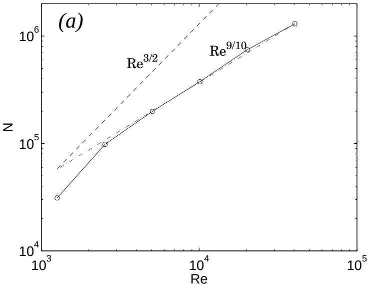


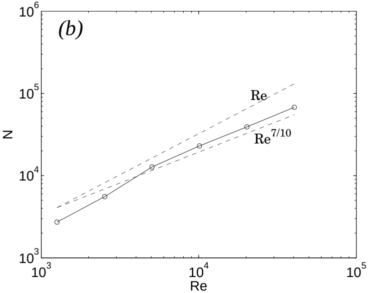
$t \in [0, 2.1]$ $t \in [123.8, 126.0]$ Spatial grid only at $t = 126.0$

Note the strong time intermittency of the solution: the smallest time step is strongly localized in space.

Scaling of modes with Reynolds number



Space–time



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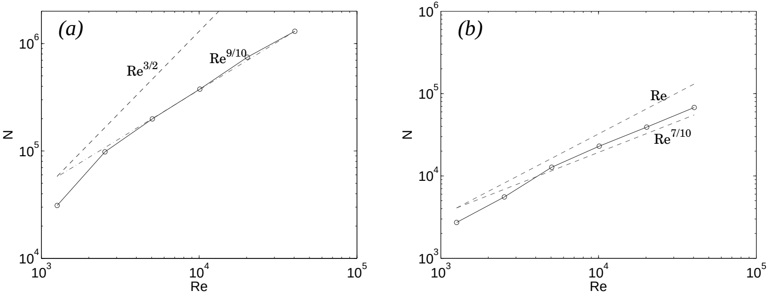

Space–time Space only

Note that intermittency reduces the number of modes significantly compared with the usual computational estimates.

β-model fractal dimension

The β-model for two-dimensional turbulence implies that the spatial modes should scale like $\mathcal{N} \sim \mathrm{Re}^{\frac{3D_F}{D_F+1}}$,

- Spatial fractal dimension is $\boxed{D_F \approx 1.2}$
- Temporal fractal dimension is $\boxed{D_F \approx 0.3}$
- Flow appears to be much more intermittent in time

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The β-model for two-dimensional turbulence implies that the spatial modes should scale like $\mathcal{N} \sim \mathrm{Re}^{\frac{3D_F}{D_F+1}}$, and temporal modes should scale like $\mathcal{N} \sim \mathrm{Re}^{\frac{3D_F}{D_F+4}}$.

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Assumes that the active proportion of the flow decreases like lengthscale to the power $2 - D_F$.

Next step: 3-D turbulence

- 3-D turbulence is more intermittent.
- Vortex stretching ("sinews" of turbulence).
- Shape and topology of vortices is complicated.
- Is 4-D space–time simulation feasible at large *Re*?
- How do we interpret 4-D structure of space–time dynamics?

$Re_\lambda = 72$, 100 times compression.

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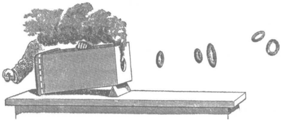
Engines of turbulence



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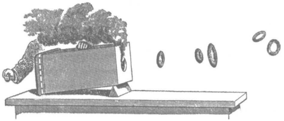
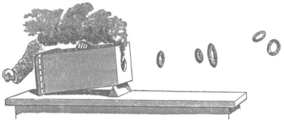


Coherent vortices are an efficient and accurate basis for turbulence simulation.

Engines of turbulence



Coherent vortices are an efficient and accurate basis for turbulence simulation.

The resulting adaptive computational modes provide insight into the dynamics and measure the intermittency of turbulence.