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ERRATUM

Homotopy self-equivalences of 4-manifolds

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Abstract We correct an error in the calculation of the braid diagram of Hambleton and Kreck (Math. Z. **248**, 147–172, 2004) for 4-manifolds with finite odd order fundamental group.

Our calculations of [1, Sect. 4] contain a mistake: we implicitly assumed that

$$\mathrm{Aut}_\bullet(B) = \mathrm{Isom}([\pi_1(M), \pi_2(M), k_M])$$

where $B = B(M)$ denotes the 2-type of a closed, oriented, smooth or topological 4-manifold M (see the proofs of Lemma 4.4 and Corollary 4.10, as well as the corresponding parts of the non-spin case on p. 170).

The homotopy self-equivalences of 2-stage Postnikov systems have been studied by a number of authors (see [2] and the references given there). The main result of Møller [2] applied to our special case is an exact sequence

$$0 \to H^2(\pi_1; \pi_2) \to \mathrm{Aut}_\bullet(B) \to \mathrm{Isom}([\pi_1, \pi_2, k_M]) \to 1$$

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whose extension class is related to the k-invariant of B. In order to correct the statements of our results, we define

$$\widetilde{\mathrm{Isom}}([\pi_1, \pi_2, k_M, s_M]) = \{\phi \in \mathrm{Aut}_\bullet(B) \mid \phi_*(c_*[M]) = c_*[M]\}$$

This is the subgroup of $\mathrm{Aut}_\bullet(B)$ fixing the image of the fundamental class of M in $H_4(B; \mathbf{Z})$. In the case $\pi_1(M)$ is finite of odd order, we obtain an exact sequence

$$0 \to H^5(\pi_1; \mathbf{Z}) \to \widetilde{\mathrm{Isom}}([\pi_1, \pi_2, k_M, s_M]) \to \mathrm{Isom}([\pi_1, \pi_2, k_M, s_M]) \to 0$$

and this middle term is the image of $\mathrm{Aut}_\bullet(M)$ in $\mathrm{Aut}_\bullet(B)$. The calculation

$$H^2(\pi_1; \pi_2) = H^5(\pi_1; \mathbf{Z})$$

is contained in the proof of Proposition 4.8. The statements of Lemma 4.4 and Corollary 4.10, together with Theorems A and B are now corrected by replacing the arithmetic group $\mathrm{Isom}([\pi_1, \pi_2, k_M, s_M])$ with its finite extension

$$\widetilde{\mathrm{Isom}}([\pi_1, \pi_2, k_M, s_M]).$$

We would also like to point out that the injectivity of the first map in the exact sequence the end of [1, p. 162] holds for odd order fundamental groups, but not in general.

References

1. Hambleton, I., Kreck, M.: Homotopy self-equivalences of 4-manifolds. Math. Z. **248**, 147–172 (2004)
2. Møller, J.M.: Self-homotopy equivalences of group cohomology spaces. J. Pure Appl. Algebra **73**, 23–37 (1991)